Nov 6-10:09 AM

4-3 day 2 The Concavity Test

Learning Objectives:

I can identify an inflection on a graph and I understand the relationship between this point and the derivatives of the function.

I can use the concavity test to find points of inflection.

I can identify the intervals on which a function is concave up or concave down.

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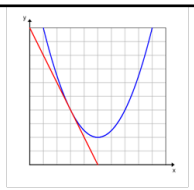
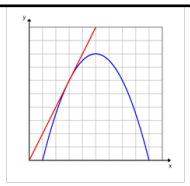


concave upwards



concave downwards

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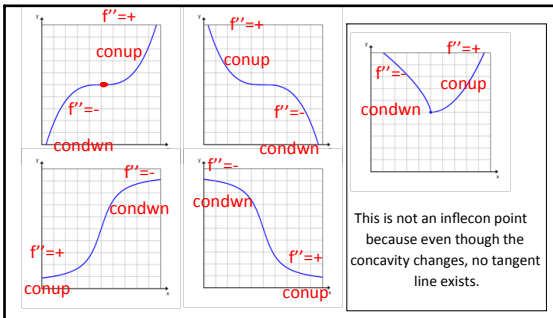

An ***inflection point*** is any point on the graph of a function where the concavity changes AND a tangent line exists.

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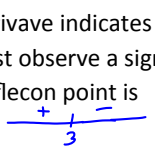
Concavity Test

f''(x)>0 f(x) is concave up

f''(x)<0 f(x) is concave down

2nd derivaves find concavity (tell us if the funcon is concave up or concave down) and are used to find inflecon points.

A sign change in the second derivative indicates a change in concavity. You must observe a sign change to be sure that an inflecon point is present.



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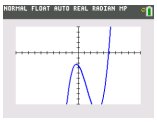
Ex1. Use the concavity test to determine the intervals on which the graph is concave up or concave down. Identify any inflection points.

1.) $f(x) = x^3 - 4x^2 - 3x - 5$

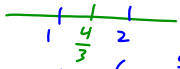
$f'(x) = 3x^2 - 8x - 3$

$f''(x) = 6x - 8 = 0$

$x = \frac{4}{3}$



$x = \frac{4}{3}$ is an inflection point because f'' is changing from neg. to pos.



conc. down $(-\infty, \frac{4}{3})$

conc. up $(\frac{4}{3}, \infty)$

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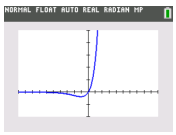
2.) $y = xe^x$

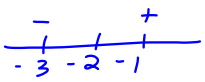
$y' = xe^x + e^x$

$y'' = xe^x + 2e^x = 0$



$e^x(x+2) = 0$

$e^x \neq 0$ $\quad x+2=0$, $x=-2$



inf. pt. at $x=-2$ because f'' changes from neg. to +

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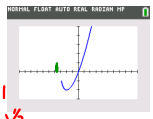
3.) $y = x\sqrt{x+3}$



$f = x$ $\quad g = (x+3)^{1/2}$

$f' = 1$ $\quad g' = \frac{1}{2}(x+3)^{-1/2}$

$y' = x \cdot \frac{1}{2}(x+3)^{-1/2} + (x+3)^{1/2}$

Concave up everywhere

$f = \frac{1}{2}x$ $\quad g = (x+3)^{-1/2}$

$f' = \frac{1}{2}$ $\quad g' = -\frac{1}{2}(x+3)^{-3/2}$

$y'' = -\frac{1}{2}x \cdot \frac{1}{2}(x+3)^{-3/2} + \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2}(x+3)^{-1/2}$

$y'' = \frac{1}{4}x(x+3)^{-3/2} + (x+3)^{-1/2} = 0$

und 0

-5 -4 -3

$-\frac{1}{4}\frac{x}{\sqrt{(x+3)^3}} = \frac{-1}{\sqrt{x+3}}$

$\frac{\sqrt{(x+3)}}{\sqrt{(x+3)^3}} = \frac{-4}{x}$

$\frac{1}{\sqrt{(x+3)^2}}$

$\frac{1}{x+3} \times \frac{4}{x}$

$x = +4x + 12$

$-3x = 12$

$x = -4$

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Homework

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